Ex Post Inefficiencies in a Property Rights Theory of the Firm[*]

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Abstract

Private information can lead to inefficient bargaining between managers. I develop a property rights theory of the firm to analyze the optimal ownership structure that minimizes this bargaining inefficiency. I first show that a change in the ownership structure that reduces the managers' aggregate disagreement payoff increases the probability that they realize efficient trades but also increases the cost of disagreement and can lead them to trade 'too often.' I then show that joint ownership is optimal if the managers' expected gains from trade are large and that either integration or non-integration is optimal if the expected gains from trade are small.

Keywords: vertical integration, property rights, asymmetric information

JEL classifications: D23, D82, L22

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1   Introduction

Economists and practitioners have long believed that ex post inefficiencies play an important role in determining the ownership structures of firms. More than 60 years ago Coase (1937) argued that the resources used to discover and haggle over the terms of trade constitute a major cost of market transactions and that these costs can be reduced if the transactions are brought into the firm.[1] Similarly, the modern transaction cost theory of the firm, as developed by Williamson (1975, 1985), argues that bargaining between firms over the sharing of quasi-rents can be costly due to the presence of private information and that these haggling costs can be reduced if the firms integrate.[2] More recently, Baker, Gibbons, and Murphy (2002) report on a series of interviews with practitioners who design the governance structures of strategic alliances. In the view of these practitioners one of the three most important factors in determining governance structures of strategic alliances is "the need for governance structures to induce efficient behavior ex post, since contracts cannot."[3]

In spite of the widely held belief that ex post inefficiencies are an important determinant of the ownership structures of firms, only few papers have formally analyzed this relationship. The literature on the property rights theory of the firm (Grossman and Hart (1985) and Hart and Moore (1990)), in particular, has so far focused on how the ownership structures of firms affect the incentives of managers to make ex ante relationship-specific investments and has largely abstracted from ex post bargaining inefficiencies. Most contributions to this literature assume that bargaining between managers is efficient and analyze how changes in the ownership structures of firms, by changing the managers' disagreement payoffs, affect the ex post bargaining outcome and thus the ex ante investment incentives.[4] In contrast, in this paper I develop a property rights theory of the firm that focuses exclusively on ex post inefficiencies and abstracts from ex ante investment inefficiencies. I argue that managerial bargaining is often hindered by private information and that the size of the resulting bargaining inefficiency depends crucially on the ownership structure of the managers' firms. I then derive the optimal ownership structures that minimize the ex post bargaining inefficiency and note

that they are frequently observed in the real world.

Suppose a seller who privately knows her cost of producing an input and a buyer who privately knows his valuation for it bargain with each other over its price. It is well known that the two 'managers' then have a strong incentive to misrepresent their valuations and that, as a result, they may fail to realize some efficient trades (Myerson and Satterthwaite (1983)). In this paper I analyze how this 'classic' bargaining inefficiency depends on the ownership structures of the managers' firms. To see how the ownership structure affects the bargaining inefficiency, suppose that the managers need some machines to engage in production and that, before learning their private information, they can contract over the ownership distribution of these machines. In the property rights approach firms are defined by their physical assets. The ownership structures of the managers' firms are therefore determined by the ownership distribution of the machines - that is they are determined by who owns what machine. The key assumption in the property rights approach, and the key assumption I adopt in this paper, is that the ownership structure that the managers agree on ex ante only determines the payoffs the managers realize if they do not agree to trade ex post. Thus, changes in the ownership structures do not affect the amount of private information that the managers have or the degree to which they act opportunistically and only affect their ex post disagreement payoffs. I first analyze how changes in the ownership structures, by changing the disagreement payoffs, affect the managers' bargaining strategies and thus the bargaining inefficiency. I then derive the optimal ownership structure which minimizes the expected bargaining inefficiency and which the managers therefore agree on ex ante. I show that, depending on the size of the expected gains from trade and the nature of the physical assets, the optimal ownership structure can be integration, non-integration, or joint ownership. Thus, the classic bargaining inefficiency that arises when managers bargain in the presence of private information is minimized by frequently observed ownership structures.

The observation that non-integration can minimize ex post inefficiencies stands in contrast to Williamson (1975, 1985). As mentioned above, he argues that vertical integration eliminates the bargaining inefficiencies that can arise when two independent

firms bargain with each other in the presence of private information.[5] I show below that bargaining between two managers can be inefficient even if they work for the same integrated firm and that vertical integration can actually increase the bargaining inefficiency. Thus, it can be the case that two managers who work for the same firm haggle more with each other, and thus waste more resources, than managers who work for two independent firms. This can be the case as long as the managers' human capital is important and they have some private information even if they work within the same firm.

Managers often adopt ownership structures that make them more dependent on each other by reducing their aggregate disagreement payoff. For instance, managers sometimes exchange 'ugly princess hostages.'[6] This refers to a practice in which managers exchange ownership of assets that are very valuable to themselves but which have little or no value to the other party. In the Japanese car industry, for instance, it can be observed that physical assets which are specific to a particular car manufacturer are sometimes owned by its supplier.[7] Also, firms sometimes reduce their aggregate disagreement payoff by selling off assets that can be used unilaterally during a conflict. For instance, after settling a costly dispute about their alliance, KLM and Northwest Airlines further increased their interdependence by eliminating their duplicate support operations.[8] It is often argued informally that managers take such actions to ensure that their trading relationships 'work well' and that conflicts are settled quickly. I show below that a change in the ownership structure that leads to a reduction in the aggregate disagreement payoff indeed reduces the probability of inefficient ex post disagreements. However, it can also induce the managers to agree 'too often' and, of course, makes them worse off if they do end up disagreeing with each other. It is therefore not immediately clear whether a change in the ownership structure that reduces the managers' aggregate disagreement payoff is in their joint interest. My analysis shows that such an ownership change does make the managers better off if their expected gains from trade are large and that it makes them worse off otherwise.

The property rights approach that I adopt in this paper, that is the assumption that changes in the ownership structures only affect the managers' disagreement pay-

offs, allows me to analyze the effects of changes in the ownership structures of firms on managerial bargaining inefficiencies without making assumptions about how such changes affect the degree of asymmetric information. This approach is different from that taken by Arrow (1975) and Riordan (1990) who also study vertical integration under imperfect information but who assume that vertical integration reduces the amount of private information that managers have. This assumption contrasts with recent papers by Aghion and Tirole (1997) and Dessein (2002) which show that the incentives of agents to transmit information to a principal may actually be reduced, and thus informational asymmetries be increased, when a principal gains more control over an agent's actions. In the absence of clear theoretic and empirical predictions about how changes in the ownership structures of firms affect the degree of asymmetric information I deliberately abstract from any such effects and instead focus exclusively on how changes in the ownership structures influence the bargaining inefficiency by determining the managers' ex post disagreement payoffs.

While most of the existing property rights literature has focused on ex ante inefficiencies, there is a small but growing literature that has allowed for, and focused on, the effect of ownership on ex post inefficiencies. Hart and Moore (1998), for instance, study the design of a firm's constitution when ex post asymmetric information prevents recontracting. Recently, Hart and Holmström (2002) developed a model in which decisions are non-contractible but can be transferred through ownership. Ownership then matters since it affects the decisions that are taken ex post. Baker, Gibbons, and Murphy (2002) analyze how ex post inefficiencies affect the design of strategic alliances, focusing on the role of relational contracts. The main difference between those papers and this one is that they rule out ex post negotiations all together while I allow for ex post negotiations which are hindered by the presence of private information. In a recent paper Hagedorn (2003) analyses optimal ownership structure of firms in the presence of both ex ante and ex post inefficiencies, where the latter are due to the presence of private information.

Finally, a paper that is similar to mine in spirit but different both in application and the economic mechanisms that it analyzes is Ayres and Talley (1995). They analyze

how divided legal entitlements can facilitate bargaining between plaintiffs and defendants. In their model divided legal entitlements can reduce the bargaining inefficiency by introducing uncertainty over which one of the parties will be the buyer and which one will be the seller. As such their analysis is closer to Cramton, Gibbons, and Klemperer (1985) than to this paper in which there is no uncertainty over the identity of the buyer and the seller.

In the next section I first describe the model. I then solve an example before analyzing the full model and discussing two extension. In Section 3 I discuss the implications and limitations of the model. Finally, I conclude in Section 4.

2   The Model

There are two risk neutral and liquidity unconstrained managers, a buyer (she) and a seller (he), and two physical assets a_1 and a_2. The assets are owned by the managers. The set of assets owned only by the buyer is denoted by $A_b \subseteq \{a_1, a_2\}$ and that owned only by the seller is denoted by $A_s \subseteq \{a_1, a_2\}$. The ownership structure is given by $A = (A_b, A_s)$.

The managers can either trade an 'input' with each other or with third parties in the market. If the managers trade with each other, they both have access to both assets, regardless of the ownership structure. The seller can then use the assets to produce the input at cost $c \in [\underline{c}, \overline{c}]$ and the buyer can use them to turn the input into a final good that she values at $\pi \in [\underline{\pi}, \overline{\pi}]$, where $\underline{c}, \overline{c}, \underline{\pi}, \overline{\pi} \in \Re$ are exogenously given parameters. It is common knowledge that the 'trade payoffs' π and c are independently drawn from distributions with density functions $f_b(\pi)$ and $f_s(c)$. I allow for three common distributions: uniform with support $[0, 1]$, standard normal, and standard exponential. Most of the results continue to hold for more general distributions, in particular, for any distribution that satisfies the monotone hazard rate condition. I discuss this generalization in Section 2.3.

If the buyer and the seller do not trade with each other, they realize their respective disagreement payoffs $b(A_b) \in \Re$ and $s(A_s) \in \Re$. Their aggregate disagreement payoff is denoted by $d(A) \equiv b(A_b) + s(A_s)$. The disagreement payoffs are given by the net payoffs

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each manager can realize by trading with third parties in the market. They depend on the ownership structure since, in the case of disagreement, only the sole owner of an asset has the right to decide how it should be used. This is the key assumption in the property rights approach which defines ownership as a 'residual control right,' i.e. right to decide how an asset should be used as long as it is not inconsistent with any prior contract.[9] Suppose, for instance, that the buyer owns one asset and the seller the other and that they fail to reach an agreement on the price of the input. Each manager can then use the asset that he or she owns to engage in production and can buy or sell inputs in the market. The disagreement payoffs differ from the payoffs the managers can realize by trading with each other for two main reasons. First, if they do not trade with each other, they only have access to the assets of which they are the sole owners while if they do trade with each other, they have access to all assets. Second, if they do not trade with each other they do not have access the human capital of the other manager. I follow the property rights approach in assuming that the disagreement payoffs are exogenously given.

Ex ante the managers contract over the ownership structure A and over the bargaining game that takes place ex post. The managers can choose any bargaining game that satisfies the balanced budget constraint. At the interim stage, the buyer learns the realization of π and the seller learns the realization of c. Each manager then decides whether or not to participate in the ex post bargaining game. If either the buyer or the seller decides not to participate in the bargaining game each manager realizes his or her respective (interim) disagreement payoff $b(A_b)$ and $s(A_s)$ and the game ends. If both managers decide to participate, then time moves on to the ex post stage at which the managers bargain over the price of the input according to the rules of the contractually specified bargaining game. If the managers agree to trade the input at price p, then the buyer's payoff is given by $\pi - p$ and the seller's is given by $p - c$. If they do not agree to trade the input, then they again realize their respective (ex post) disagreement payoffs $b(A_b)$ and $s(A_s)$.

For most of the analysis I assume that the ownership structures are deterministic and cannot be made contingent on information that is revealed after the ex ante contracting

stage. I believe that in many situations this is a reasonable assumption to make and I discuss the implications of relaxing it in Section 2.4.

I denote the ownership structure that minimizes the aggregate disagreement payoff by $\underline{A} \equiv \arg\min_A d(A)$ and the ownership structure that maximizes the aggregate disagreement payoff by $\overline{A} \equiv \arg\max_A d(A)$. The corresponding aggregate disagreement payoffs are denoted by $\underline{d} \equiv d(\underline{A})$ and $\overline{d} \equiv d(\overline{A})$. Also, I assume that the bounds on the aggregate disagreement payoffs satisfy $\underline{d} > \underline{\pi} - \overline{c}$ and $\overline{d} < \overline{\pi} - \underline{c}$. I make these assumptions only because they simplify the exposition. It would be trivial to analyze the implications of relaxing them.[10]

I refer to an ownership structure in which the buyer owns both assets as 'buyer integration' and define 'seller integration' accordingly. Under 'non-integration' each manager owns one asset and under 'joint ownership' both managers own both assets. Finally, I refer to $E_{c,\pi}\left[\pi - c - \overline{d} \mid \pi - c - \overline{d} \geq 0\right]$ as the managers' 'expected gains from trade.'[11]

This model is closely related to that analyzed in Myerson and Satterthwaite (1983). They analyze a bilateral bargaining situation with two-sided asymmetric information and derive the optimal direct bargaining mechanism that maximizes the expected gains from trade. I differ from their analysis in that I do not take the managers' disagreement payoffs as given and instead allow them to contract over them and also by restricting myself to uniform, standard normal, and standard exponential distributions. In spite of this difference I can draw extensively on their results in solving the model.

2.1 An Example

Before solving the full model it is instructive to analyze the following example which illustrates most of the intuition. Suppose that the trade payoffs π and c are uniformly distributed on $[0, 1]$ and that the bargaining game that the managers play is exogenously given. In particular, suppose that at the ex post bargaining stage the managers play a double auction: the buyer and the seller respectively submit sealed bids $p_b \in \Re$ and $p_s \in \Re$. If $p_b \geq p_s$, trade takes place at price $p = \frac{1}{2}(p_b + p_s)$ and if $p_b < p_s$, trade does not take place. Moreover, suppose that managers' bargaining strategies are linear

functions of their trade-offs.[12]

I solve this example by backward induction. The bargaining strategies that the managers adopt at the ex post stage are described in the following proposition.

PROPOSITION 1. The strategies

$$
p_b(\pi) = \begin{cases} \frac{1}{12}\left(1 - 9b(A_b) + 3s(A_s)\right) + \frac{2}{3}\pi & \text{if } \pi \leq \frac{1}{4}(5 + 3d(A)) \\ \\ \frac{1}{12}(11 - 3b(A_b) + 9s(A_s)) & \text{if } \pi > \frac{1}{4}(5 + 3d(A)) \end{cases}
$$

and

$$
p_s(c) = \begin{cases} \frac{1}{12}(3 - 3b(A_b) + 9s(A_s)) + \frac{2}{3}c & \text{if } c \geq -\frac{1}{4}(1 + 3d(A)) \\ \\ \frac{1}{12}\left(1 - 9b(A_b) + 3s(A_s)\right) & \text{if } c < -\frac{1}{4}(1 + 3d(A)), \end{cases}
$$

form a linear Bayesian Nash equilibrium of the double auction.

Proof of Proposition 1. See the Appendix.

It is important to observe that for any $d(A) \in (-1, 1)$ this equilibrium is ex post inefficient. To see this, note that trade takes place if and only if $p_b(\pi) \geq p_s(c)$. Substituting the expressions for $p_b(\pi)$ and $p_s(c)$ from Proposition 1 into this inequality and rearranging shows that trade takes place if and only if

$$
\pi - c \geq d(A) + \frac{1}{4}(1 - d(A)).
$$

Thus, ex post efficient trading opportunities are not realized for any $\pi - c \in (d(A), d(A) + \frac{1}{4}(1 - d(A)))$.

At the interim stage, after learning the realizations of their trade payoffs, the managers always choose to participate in the ex post bargaining game since doing so ensures them a payoff that is at least as large as their disagreement payoffs. Ex ante the managers then contract over the ownership structure A. Since there is no private information at the contracting stage, I assume that bargaining over the ownership structure is efficient. The managers, who are risk neutral and not wealth constrained, then agree on the ownership structure that maximizes social welfare, independent of its ex post distribution. Formally, the managers choose the optimal ownership structure that solves

$$\max_{A} W(d(A)), \qquad (1)$$

where $W\left(d\left(A\right)\right)$ denotes social welfare and is given by

$$W(d\left(A\right)) = d\left(A\right) + E_{c,\pi}\left[(\pi - c - d\left(A\right))q(c,\pi,d(A))\right], \qquad (2)$$

and

$$q\left(c,\pi,d(A)\right) = \begin{cases} 1 & \text{if } \pi - c \geq d\left(A\right) + \frac{1}{4}(1 - d\left(A\right)) \\ \\ 0 & \text{otherwise.}^{13} \end{cases}$$

Note that social welfare depends on the aggregate disagreement payoff $d(A)$ and not on the distribution of the individual disagreement payoffs $b(A_b)$ and $s(A_s)$. Therefore a change in the ownership structure only affects efficiency if it alters the aggregate disagreement payoff and not if it simply leads to a redistribution of disagreement payoffs from one manager to the other.

The key to solving the contracting problem (1) is to understand how a change in the aggregate disagreement payoff affects social welfare. Suppose the managers consider a change in the ownership structure from $\overline{A}$ to A' and that this leads to a reduction in the aggregate disagreement payoff from $\overline{d}$ to $d' \equiv d(A') < \overline{d}$. The effect on social welfare of such a change is given by

$$W(d') - W(\overline{d}) = -(\overline{d} - d')\left(1 - q(c,\pi,d')\right) + \qquad (3)$$
$$E_{c,\pi}\left[(\pi - c - \overline{d})(q(c,\pi,d') - q(c,\pi,\overline{d}))\right].$$

On the one hand, the managers realize a lower aggregate payoff if they disagree even after the ownership change. This effect is always negative and is captured by the first term on the RHS of (3). On the other hand, however, a reduction in the aggregate disagreement payoff also commits at least one of the managers to a more cautious bargaining strategy. This increases the probability that trade takes place ex post. In particular, for any realizations of π and c that satisfy $\pi - c \in \left(\frac{3}{4}d' + \frac{1}{4}, \frac{3}{4}\overline{d} + \frac{1}{4}\right]$ trade will take place only after the ownership change. The second term on the RHS of (3) captures

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the welfare implication of the increase in the probability of trade. Its sign is ambiguous and depends crucially on the size of the reduction in the aggregate disagreement payoff. If the reduction is small, in the sense that $\frac{3}{4}d' + \frac{1}{4} \geq \overline{d}$, the gains $\pi - c$ from each of the additional trades are larger than the aggregate disagreement payoff $\overline{d}$. Thus, in this case the increase in the probability of trade is unambiguously welfare improving. For larger reductions in the aggregate disagreement payoff, however, some of the additional trades the managers realize are 'ex ante inefficient,' in the sense that they satisfy $\pi - c < \overline{d}$. While these trades are ex post efficient given d', the managers would be better off realizing the maximum aggregate disagreement payoff than engaging in these trades. Thus, for large reductions in the aggregate disagreement payoff, that is for $\frac{3}{4}d' + \frac{1}{4} < \overline{d}$, the increased probability of trade has an ambiguous effect on social welfare.

Faced with this trade-off, what ownership structure do the managers choose? It turns out the answer to this question is surprisingly straightforward:

PROPOSITION 2. At the ex ante stage it is weakly optimal for the managers to agree on the ownership structure

$$A^* = \begin{cases} \overline{A} & \text{if } \overline{d} \geq d_{crit} \\ \underline{A} & \text{otherwise,} \end{cases}$$

where $d_{crit} \in (0, 1)$.

Proof of Proposition 2. Let $d \in [-1, 1]$ and consider the social welfare function $W(d)$ as defined in (2). Figure 1 illustrates how $W(d)$ varies with d.[14] The proposition follows from two properties of $W(d)$: its only interior extremum is a minimum (at $\widehat{d} = -7/9$) and it reaches its global maximum at $d = 1$. We can then distinguish two cases. First, suppose $\underline{d} < \widehat{d}$. Then d_{crit} is implicitly defined by the unique $d \in (\underline{d}, 1)$ that solves $W(d) = W(\underline{d})$. Second, suppose $\underline{d} \geq \widehat{d}$. Then $\overline{A}$ is always optimal, i.e. $d_{crit} = \underline{d}$. ∎

Thus the managers optimally choose the ownership structure that maximizes their aggregate disagreement payoff if, for a given distribution of trade payoffs, the expected gains from trade are small (i.e. when $\overline{d}$ is large) and they choose the ownership structure that minimizes the aggregate disagreement payoff otherwise. The intuition for this proposition is straightforward. Recall that a marginal increase in d increases the payoff

the managers realize in the case of disagreement but also reduces the probability that efficient trades take place.[15] If d is small, disagreement does not occur very often and thus the benefit of a marginal increase in d is quite small and is indeed dominated the cost. The larger d, however, the more likely it is that disagreement occurs and thus the larger the benefit of a further in marginal increase in d. For d large enough, the benefit of a further marginal increase in d eventually dominates the cost. The property that a marginal increase in d is welfare reducing for small d and welfare enhancing for large d is the key reason behind the main results of the example as summarized in Proposition 2.

2.2   The Analysis

I now return to the full model described at the beginning of Section 2. The Revelation Principle (see, for example, Myerson (1991)) allows me to restrict the analysis, without loss of generality, to Bayesian incentive compatible direct mechanisms. Suppose then that, after having learned their trade payoffs, the buyer and the seller make respective announcements $\hat{\pi}$ and $\hat{c}$ of their trade payoffs. A direct mechanism specifies the probability of trade $q(\hat{c}, \hat{\pi})$ and expected price of good $t(\hat{c}, \hat{\pi})$ as a function of these announcements. At the ex ante contracting stage the managers then agree on the optimal trading rule $q^*(\cdot)$, the optimal transfer rule $t^*(\cdot)$, and the optimal ownership structure A^* that maximize social welfare

$$W(d(A), \ q(c, \pi)) = d(A) + E_{\pi, c} \left[(\pi - c - d(A)) q(c, \pi) \right]$$

subject to the incentive compatibility and interim individual rationality constraints.[16]

The ex ante contracting problem can be separated into two parts: first solve for the optimal trading and transfer rules for any given ownership structure and then, second, solve for the optimal ownership structure. The first part - finding the optimal mechanism $(q^*(\cdot), t^*(\cdot))$ - was solved by Myerson and Satterthwaite (1983) and the optimal trading rule that they derive is described in the following lemma.

LEMMA 1.   The optimal trading rule $q^*(\cdot)$ that maximizes $W(d(A), q(c, \pi, d(A)))$ for given disagreement payoffs $d(A)$ is given by

$$q(c, \pi, d(A)) = \begin{cases} 1 & \text{if } \pi - c - d(A) \geq \frac{\lambda(d(A))}{1+\lambda(d(A))} \left(\frac{1-F_b(\pi)}{f_b(\pi)} + \frac{F_s(c)}{f_s(c)} \right) \\ 0 & \text{otherwise,} \end{cases}$$

where $\lambda(d(A)) \in (0, \infty)$ solves the interim participation constraint

$$E_{c,\pi} \left[\left(\pi - c - d(A) - \frac{1-F_b(\pi)}{f_b(\pi)} - \frac{F_s(c)}{f_s(c)} \right) q(c, \pi, d(A)) \right] = 0. \qquad \text{(IR)}$$

Proof of Lemma 1. This lemma follows immediately from Myerson and Satterthwaite (1983) and I refer to their analysis. ■

Note that since $\lambda(d(A))$ is strictly positive, bargaining is inefficient even if the managers play the most efficient bargaining game. In particular, just as in the example, the managers sometimes do not trade although it would be efficient for them to do so. At the ex ante contracting stage the managers agree on the ownership structure that minimizes this well-known bargaining inefficiency. Thus, they solve

$$\max_A W(d(A), \; q(c, \pi, d(A))). \qquad (4)$$

Note that social welfare again only depends on the aggregate disagreement payoff and not on the distribution of the individual disagreement payoffs. The effect of a change in the ownership structure on social welfare is also very similar to that described in the example. To see this suppose that the managers consider a change from $\overline{A}$ to A' with a corresponding reduction in the aggregate disagreement payoff from $\overline{d}$ to $d' < \overline{d}$. The effect on social welfare is then given by

$$W(d', q(c, \pi, d')) - W(\overline{d}, q(c, \pi, \overline{d})) =$$

$$(5)$$

$$-(\overline{d} - d')E_{c,\pi}\left[1 - q(c, \pi, d')\right] + E_{c,\pi}\left[(\pi - c - \overline{d})(q(c, \pi, d')) - q(c, \pi, \overline{d}))\right].$$

The first term on the RHS captures the welfare reduction that is due to the lower payoff the managers realize in case they still disagree after the change in the ownership structure and the second term on the RHS captures the effect on welfare that is due to the change in the probability of trade. A reduction in the aggregate disagreement payoff increases the probability that the managers trade ex post since a lower aggregate

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disagreement payoff relaxes the participation constraint (see (IR)) and thus increases the set of feasible trading rules from which the managers can choose at the contracting stage. The welfare effect of the increase in the probability of trade is again ambiguous: while small reductions in the aggregate disagreement payoff only increase the probability that the managers realize ex ante efficient trades, large reductions in the aggregate disagreement payoff may induce them to also realize trades that are ex ante inefficient.

To understand what ownership structure optimizes this trade-off and thus maximizes social welfare, consider the following proposition.

PROPOSITION 3.   The optimal ownership structure is given by

$$A^* = \begin{cases} \overline{A} & \text{if } \overline{d} \geq d_{crit} \\ \underline{A} & \text{otherwise,} \end{cases}$$

where $d_{crit} \in (\underline{\pi} - \overline{c}, \overline{\pi} - \underline{c})$.

Proof of Proposition 3. I use Matlab to numerically solve $W(d, q(c, \pi, d))$ for different values of d. In Figure 1 I plot the social welfare function for the case when the trade payoffs are uniformly distributed on $[0, 1]$ and in Figure 2 I provide a representative sketch of the social welfare functions when the distributions of the trade payoffs are given by the standard normal or the standard exponential distribution. The functions all share two key properties: they are quasi-convex and reach their global maximum as $d \to \overline{\pi} - \underline{c}$. The proposition follows immediately from these properties. In particular, let $\widehat{d}$ be the $d \in [\underline{\pi} - \overline{c}, \overline{\pi} - \underline{c})$ at which $W(d, q(c, \pi, d))$ is minimized. We can then distinguish two cases. First, suppose $\underline{d} < \widehat{d}$. Then d_{crit} is implicitly defined by the unique $d \in (\underline{d}, \overline{d})$ that solves $W(d) = W(\underline{d})$. Second, suppose $\underline{d} \geq \widehat{d}$. Then $\overline{A}$ is always optimal, i.e. $d_{crit} = \underline{d}$. ∎

Thus, just as in the example, the managers optimally choose the ownership structure that maximizes their aggregate disagreement payoff if, for a given distribution of trade payoffs, the expected gains from trade are small (i.e. when $\overline{d}$ is large) and they choose the ownership structure that minimizes the aggregate disagreement payoff otherwise. Note that Proposition 3 does not rule out the possibility that $\overline{A}$ is always optimal, i.e. $d_{crit} = \underline{d}$ for all $\underline{d} \in (\underline{\pi} - \overline{c}, \overline{\pi} - \underline{c})$. Indeed, this is the case when the distributions of the trade payoffs are given by the standard exponential distributions in which case the

social welfare function is monotonically increasing in d (see Figure 2).[17]

2.3 More General Distributions

In the analysis above I have restricted attention to uniform, standard normal, and standard exponential distributions. In this subsection I extend the model by allowing for more general distributions. Suppose that the trade payoffs π and c are independently drawn from distributions with continuous and strictly positive density functions $f_b(\pi)$ and $f_s(c)$ and corresponding cumulative density functions $F_b(\pi)$ and $F_s(c)$ that satisfy the monotone hazard rate conditions

$$\frac{d}{d\pi}\left(\frac{f_b(\pi)}{1 - F_b(\pi)}\right) \geq 0, \ \forall \pi \in [\underline{\pi}, \overline{\pi}], \quad \text{and} \quad \frac{d}{dc}\left(\frac{f_s(c)}{F_s(c)}\right) \leq 0, \ \forall c \in [\underline{c}, \overline{c}].$$

In this case Lemma 1 still holds so that optimal ownership structure is still given by the solution to (4) and the trade-off that the managers face when deciding on the ownership structure is still given by (5). The optimal ownership structure on which the managers agree is described in the following proposition.

PROPOSITION 4. The optimal ownership structure is given by

$$A^* = \begin{cases} \overline{A} & \text{if } d(\overline{A}) \geq d_{crit} \\ A(F_b(\cdot), F_s(\cdot)) & \text{otherwise,} \end{cases}$$

where $d_{crit} \in [\underline{\pi} - \overline{c}, \overline{\pi} - \underline{c})$ and $A(F_b(\cdot), F_s(\cdot))$ can be any ownership structure.

Proof of Proposition 4. It follows from Lemma 1 that $W(d, q(c, \pi, d)) \to \overline{\pi} - \underline{c}$ as $d \to \overline{\pi} - \underline{c}$. Since $\pi - c \leq \overline{\pi} - \underline{c}$ for all $\pi \in [\underline{\pi}, \overline{\pi}]$ and $c \in [\underline{c}, \overline{c}]$ this implies that $W(d, q(c, \pi, d))$ reaches its global maximum as $d \to \overline{\pi} - \underline{c}$. Thus, there exists a $d_{crit} < \overline{\pi} - \underline{c}$ such that $W(d, q(c, \pi, d))$ is maximized at $\overline{d}$ for all $\overline{d} \geq d_{crit}$. It follows that $A^* = \overline{A}$ if $\overline{d} \geq d_{crit}$. I cannot characterize the shape of $W(d, q(c, \pi, d))$ for $d < d_{crit}$ so that, in principle, any ownership structure $A(F_b(\cdot), F_s(\cdot))$ can be optimal if $\overline{d} < d_{crit}$. ∎

It is therefore still the case that the managers choose $\overline{A}$ if the gains from trade are small, i.e. when $\overline{d}$ is large. Also, when the gains from trade are large they may still want to choose an ownership structure that reduces the aggregate disagreement payoff. However, I have not been able to prove that minimizing the aggregate disagreement

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payoff is always optimal in this case.[18] Thus, by how much the managers reduce the aggregate disagreement payoff if the gains from trade are large may depend on the precise form of the distribution.

2.4 Mechanism Design

So far I have restricted attention to non-contingent ownership structures. In principle, the managers could, of course, agree on a direct mechanism that makes the trading- and transfer rules *and* the ownership structures dependent on their interim announcements. In this case the results from the model would no longer hold. The managers could then always relax the interim participation constraint by committing to very low interim disagreement payoffs and thus achieve first best efficiency. This is the case since, without an interim participation constraint, first best efficiency can be achieved in a bilateral bargaining situation with two-sided asymmetric information (see d'Aspremont-Gérard-Varet (1979)).

For the results of the model to hold the interim and the ex post disagreement payoffs have to be 'linked,' in the sense that the former cannot be changed without also changing the latter. This is the case if ownership structures are deterministic. With contingent ownership structures, however, the interim- and ex post disagreement payoffs are not linked. The managers' choice of the optimal ownership structure is then not determined by the trade-off between a higher cost and a lower probability of disagreement that is central in the model.

In practice, I think that it may often be difficult for managers to get a court to enforce different ownership structures at the ex post and the interim stage. While a court can observe whether the managers disagree, it may not be able to observe whether their disagreement arose at the interim or the ex post stage. In other words, the court may not be able to determine whether the managers have actually bargained with each other over the sharing of possible gains from trade.

3 Discussion

The ownership structures that the model predicts are widely observed in the real world. To see this, suppose, without loss of generality, that a_1 is more useful to the buyer than a_2 and that a_2 is more useful to the seller than a_1. Thus,

$$b(\{a_1\}) + s(\{a_2\}) \geq b(\{a_2\}) + s(\{a_1\}).$$

Furthermore, consider the following definitions:

DEFINITION 1.   The assets a_1 and a_2 are 'non-synergistic' if and only if

$$b(\{a_1\}) + s(\{a_2\}) \geq \max\left[b(\{a_1, a_2\}) + s(\emptyset), b(\emptyset) + s(\{a_1, a_2\})\right].$$

DEFINITION 2.   The assets a_1 and a_2 are 'buyer-synergistic' if and only if

$$b(\{a_1, a_2\}) + s(\emptyset) \geq \max\left[b(\{a_1\}) + s(\{a_2\}), b(\emptyset) + s(\{a_1, a_2\})\right].$$

DEFINITION 3.   The assets a_1 and a_2 are 'seller-synergistic' if and only if

$$b(\emptyset) + s(\{a_1, a_2\}) \geq \max\left[b(\{a_1\}) + s(\{a_2\}), b(\{a_1, a_2\}) + s(\emptyset)\right].$$

Hence, assets are buyer-synergistic (seller-synergistic) if, in the case of disagreement, the aggregate payoff is higher under buyer-integration (seller-integration) than under seller- (buyer-) or non-integration.   The assets are non-synergistic if, in the case of disagreement, the aggregate payoff is higher under non-integration than under either buyer- or seller-integration.

Note that aggregate disagreement payoff is minimized under joint ownership, i.e. when both managers own both assets.   Under joint ownership neither manager can use the assets unilaterally to engage in production since, when both are owners, the usage of the assets has to be approved by both.[19]   Thus,

$$b(\{a_1, a_2\}) + s(\{a_1, a_2\}) \leq b(A_b) + s(A_s), \quad \forall A_b, A_s \subset \{a_1, a_2\}.$$

In some special cases there may be other ownership structures that also minimize the aggregate disagreement payoff.   Suppose, for instance, that the assets are 'strictly

synergistic,' in the sense that $b(\{a_1\}) + s(\{a_2\}) = b(\emptyset) + s(\emptyset)$. In this case asset a_i for $i = 1, 2$ is only useful to a manager if he or she also owns asset a_j for $j \neq i$. The aggregate disagreement payoff can then be minimized by agreeing either on joint or on separate ownership of the assets. Thus, joint ownership always minimizes the aggregate disagreement payoff and in most cases it is the unique minimizer. To simplify the exposition I assume below that whenever the managers want to minimize the aggregate disagreement payoff they agree on the joint ownership of the assets.

It then immediately follows from Proposition 3 that the optimal ownership structure is given by

$$
A^* = \begin{cases}
\text{joint ownership} & \text{if } \overline{d} < d_{crit} \\
\text{buyer-integration} & \text{if } \overline{d} \geq d_{crit} \text{ and assets are buyer-synergistic} \\
\text{seller-integration} & \text{if } \overline{d} \geq d_{crit} \text{ and assets are seller-synergistic} \\
\text{non-integration} & \text{if } \overline{d} \geq d_{crit} \text{ and assets are non-synergistic}
\end{cases}
$$

Thus, joint ownership is optimal when, for a given distribution of trade payoffs, the expected gains from trade are large, i.e. when $\overline{d}$ is small. This is true independent of whether the assets are synergistic or non-synergistic. Intuitively, when the expected gains from trade are large, the managers do not disagree very often. Thus the welfare costs of reducing the aggregate disagreement payoff by moving to joint ownership are quite small and are indeed dominated by the welfare gain.

In the basic property rights model with ex ante investments joint ownership cannot be optimal (see Hart (1995)). Since joint ownership arrangements are, however, frequently observed in the real world it is important to develop theoretic arguments that might explain their existence. A number of recent papers have extended the basic property rights model and shown that under certain conditions joint ownership can provide optimal investment incentives.[20] This paper suggests an additional reason for why joint ownership can be optimal, namely that it can minimize ex post bargaining inefficiencies.

Joint ownership is not optimal when, for a given distribution of trade payoffs, the expected gains from trade are small, i.e. when $\overline{d}$ is large. Instead, in this case integration is optimal if the assets are synergistic and non-integration is optimal if the assets are

non-synergistic. When the expected gains from trade are small, the managers disagree very often. The welfare cost of a reduction in the aggregate disagreement payoff is therefore quite large and dominates the welfare benefit.

The property rights literature has been criticized for not predicting asset clusters often enough (see Holmström (1999)). Asset clusters are, of course, observed very often since most firms own large numbers of assets while their workers typically have no ownership rights over the assets they use in the production process. In a recent paper Holmström (1999, p.88) asks: "So why do firms own essentially all the nonhuman assets they use in production? Why do workers - or for that matter any other stakeholder - rarely own any such assets? This strikes me as one of the most basic regularities that a theory of the firm needs to explain." In my model asset clusters arise when assets are synergistic and, for a given distribution of trade payoffs, the expected gains from trade are small. In this case the managers know that they will disagree very often and simply want to ensure that they realize as high a payoff as possible whenever they do disagree. When assets are synergistic this is achieved by clustering the ownership rights of all assets and giving them to the party that has the highest outside value for the assets.

My result that asset clusters can minimize the bargaining inefficiency contrasts, at least superficially, with the results in Cramton, Gibbons, and Klemperer (1987). They extend Myerson and Satterthwaite (1983) by allowing for more general ownership arrangements over the good that is to be traded (and also by allowing for more than two players). They show that the bargaining inefficiency can often be eliminated if the good is jointly owned by the players. Essentially, when the good is jointly owned the players are uncertain whether they will be the buyer or the seller of the good and this reduces their incentive to exaggerate their valuation. In contrast to their analysis I maintain the assumption in Myerson and Satterthwaite (1983) that the seller initially owns the good but allow for different ownership arrangements over the assets that are needed to produce the good in the first place. Thus, in my model joint ownership affects the bargaining inefficiency not because it introduces uncertainty over the identity of the buyer and the seller but because it affects the cost of disagreement. It is for this reason that asset clustering rather than joint ownership can be optimal in my model while joint

ownership is often 'optimal' in Cramton, Gibbons, and Klemperer (1987).

It is reassuring that the ownership structures that are predicted by the model can be commonly observed. However, it of course remains an open empirical question whether they actually arise under the conditions predicted by the model. Note, though, that since the predictions of my model only depend on the level of the expected gains from trade and the nature of the productive assets, and not on marginal investment incentives, they should be easier to test empirically than the predictions of the existing property rights models.[21]

Having described the main implications and results of the model, I now turn to a discussion of its key assumptions. I start with the information structure which contains a number of assumptions. First, I assume that changes in the ownership structure do not affect the amount of private information that the managers have. As mentioned in the introduction, I make this assumption since we lack clear theoretic and empirical predictions about how, or if at all, changes in the ownership structures of firms affect the amount of private information that managers possess. One aim of this paper is to show that, even if changes in the ownership structures do not affect the amount of private information, they still have an important effect on the efficiency of ex post bargaining by determining the managers' disagreement payoffs. Second, I assume that ex ante the managers have symmetric information. This assumption allows me to focus on one inefficiency, namely the ex post bargaining inefficiency. It seems reasonable to believe that the managers may already have private information at the ex ante contracting stage and that this private information affects the ownership structures that the managers agree on. I leave the analysis of this issue for future research. Third, I assume that the managers' disagreement payoffs are common knowledge. This is not a critical restriction. In the working paper version of this paper (Matouschek (2002)) I show that the same results as above can be derived in a model in which the disagreement payoffs, rather than the trade payoffs, are uncertain. This is the case if changes in the disagreement payoffs lead to shifts of the distributions of disagreement payoffs without changing their 'shape.' Thus, what is important for the above results to hold, is that there is uncertainty over the managers' reservation prices and not whether this uncertainty comes through the

trade or the disagreement payoffs.

Another key assumption is the managers' ability to commit themselves ex ante not to renegotiate inefficient ownership structures ex post. It is clear that if costless renegotiation of ownership structures were possible, the above analysis would not continue to hold. In this case the managers would anticipate costless renegotiation when they bargain over the price of the input and would always perceive the aggregate disagreement payoff to be given by $d(\overline{A})$, regardless of the contractually agreed ownership structure. I conjecture, however, that the key results of the model would continue to hold if renegotiation were possible but costly. There are a number of reasons for why renegotiation may be costly. For instance, some ownership structures may involve the destruction of physical assets or the sale of such assets to third parties. While such ownership changes can be reversed, this can only be done at an ex post cost. Also, some ownership structures may involve (possibly contractible) specific investments, such as the location of a new plant close to an existing plant. Again, such ownership structures could only be reversed at a cost. Finally, renegotiation over the ownership structure would be costly if the managers had private information about how much they value the assets outside of the relationship. As mentioned above, I show in Matouschek (2002) that the results of this model continue to hold in a model in which there is uncertainty over the disagreement rather than the trade payoffs. While I do not allow for renegotiation even in that extension, it is clear that any such renegotiation would be costly.

Finally, I do not allow for renegotiation of the bargaining game to which the managers commit ex ante. In particular, I assume that if, after having played the contractually specified bargaining game, the managers do not agree on the price of the input then they realize their disagreement payoffs even if the buyer's valuation is larger than the seller's cost. If the managers were unable to commit not to renegotiate, the bargaining inefficiency would necessarily be larger. The result above therefore shows that the upper envelope of the social welfare function can be decreasing in the aggregate disagreement payoff. One way to study the implications of allowing the managers to make offers until all gains from trade are realized is to study an explicit bargaining game (see, for instance, Myerson (1991)). In Matouschek (2000) I analyze a dynamic bargaining game in which

the managers alternate in making offers and the ownership structure determines their inside options, i.e. the payoffs they realize while they temporarily disagree.[22] There a reduction in the aggregate inside option accelerates agreement while making temporary disagreements more costly. The results are essentially the same as those presented above.

4   Applications and Conclusions

There are many situations in which players bargain over the sharing of possible gains from trade in the presence of private information. In this paper I have shown that, in such a situation, the players may have an incentive to take actions prior to the bargaining stage to reduce their aggregate disagreement payoff. Such a reduction increases the probability that the players reach efficient agreements but also increases the costs of disagreements and might even induce them to agree 'too often.' I have shown that it is optimal for players to minimize the aggregate disagreement payoff if the expected gains from trade are large and to maximize their aggregate disagreement payoff otherwise.

I believe that this analysis can be applied beyond the ownership structure of firms, as I have done above, and might also be applicable to other institutions and contractual arrangements. An obvious, but possibly controversial, example is the marriage contract which reduces the aggregate disagreement payoff of a couple by giving veto rights over certain actions to both parties. Such a contract is typically signed by two people who anticipate being locked-in in the future and who might reasonably expect future bargaining inefficiencies due to presence of private information. In this interpretation the marriage contract is a means of facilitating domestic decision making, albeit one that comes at the cost of lower payoffs in the case of potential disagreements.

Another potential application, and one that is more closely related to the theory of the firm, is the optimal design of bankruptcy procedures (for an introduction see, for instance, Hart (1995)). Bankruptcy procedures which put a hold on the claims of creditors and allow the incumbent management a period of time to reorganize their enterprise (such as Chapter 11 in the US) have been criticized for being cumbersome and time consuming. The arguments in this paper suggest that it might be possible

to accelerate such bankruptcy procedures by reducing the aggregate payoff the parties realize during the negotiations, for example by limiting the business transactions the management is allowed to perform, and that such a change can reduce ex post inefficiencies that are due to private information. The analysis also seems applicable to the optimal design of strike legislation. In this context one could ask, for instance, if a firm should commit itself contractually not to use temporary replacement workers or not to run down inventories during strikes.

Finally, to the extent that the model can be extended to multilateral bargaining situations, arguments similar to those presented above might also be used to explain the institution that first motivated me to write this paper. In the Roman Catholic Church a new pope is elected by an assembly of cardinals who are locked up in a part of Vatican Palace until they reach an agreement. This institution, called a 'conclave,' originated in the 13th century when the cardinals failed to elect a new pope for two years. A local magistrate then decided to improve the cardinals' incentives to reach an agreement by making disagreement more uncomfortable for them. For this purpose he locked them up in the episcopal palace, removed its roof, and allowed them nothing but bread and water until they elected the next pope.[23] The observation that this institution has not been abandoned, and only somewhat adapted, suggests it might be efficient for the church as a whole, including the decision making cardinals, to accelerate the decision making process by lowering the payoff the cardinals realize during their negotiations.

While these examples suggest possible applications of the model developed above, they also raise an important final question, namely why, in my model, the managers cannot use contracts to manipulate their disagreement payoffs. Indeed, managers do sign contracts, such as exclusive supplier contracts, that reduce their disagreement payoffs and they might do so, at least partially, to facilitate ex post bargaining. I believe that the question of when managers use contracts and when they use ownership structures to manipulate their disagreement payoffs is an interesting and important one. However, I leave this question for future research since we currently seem to lack a commonly accepted framework of partially incomplete contracts that would be necessary to fruitfully investigate this question.

5 Notes

1. Coase (1937, p.390 - 391).

2. See, for instance, Williamson (1975, p.31-37).

3. Baker, Gibbons, and Murphy (2002, p.1).

4. For an overview of the property rights literature see Hart (1995).

5. See Williamson (1975, p.31-37).

6. See, for example, Williamson (1985) and, in the context of military conflicts, Schelling (1960).

7. Holmström and Roberts (1998, p.80-81).

8. Holmström and Roberts (1998, p.84).

9. See, for instance, Hart (1995, p.30).

10. These assumptions simply ensure that, at least sometimes, trade between the managers is (weakly) optimal and, at least sometimes, not trading is (weakly) optimal.

11. Note that, given this definition, the expected gains of trade depend on $\overline{d}$. I am therefore referring to the minimum expected gains from trade that are due only to the technological environment that the managers face and are not 'artificially' increased by ownership structures that reduce the aggregate disagreement payoff.

12. It is well known (see Chatterjee and Samuelson (1983)) that the double auction can have many Bayesian Nash equilibria. I focus on the linear one because of its simplicity and because Myerson and Satterthwaite (1983) have shown that if the players' reservation prices are uniformly distributed on $[0, 1]$ it is the most efficient Bayesian Nash equilibrium.

13. Social welfare is the sum of the buyer's and the seller's expected payoff. It is given by $W(d(A)) = d(A)(1 - E_{c,\pi}[q(c, \pi, d(A))]) + E_{c,\pi}[(\pi - c)q(c, \pi, d(A))] = d(A) + E_{c,\pi}[(\pi - c - d(A))q(c, \pi, d(A))]$ since they realize $d(A)$ if they do not trade, which happens with probability $(1 - q(c, \pi, d(A)))$, and they realize $\pi - c$ when they do trade, which happens with probability $q(c, \pi, d(A))$.

14. Solving (2) it is straightforward to show that $W(d) = (1 + 3d)(9 + 10d - 3d^2)/64$ if $-1/3 \leq d \leq 1$ and $W(d) = (1 + 3d)(5 + 3d)^2/192$ if $-1 \leq d \leq -1/3$.

15. Above I showed that another benefit associated with an increase in d is the reduction in the probability that managers realize ex ante inefficient trades. Note that this second benefit effect does not operate on the margin, i.e. it is only realized for sufficiently large discrete changes in d.

16. The incentive compatibility constraints ensure that each manager finds it optimal to make truthful announcements of his or her type and the interim individual rationality constraints ensure that, after learning their type, the managers prefer participating in the bargaining game to realizing the disagreement payoffs.

17. It is straightforward to show that $\underline{A}$ can be optimal (i.e. $d_{crit} > \underline{d}$ for some $\underline{d} \in (\underline{\pi} - \overline{c}, \overline{\pi} - \underline{c})$) when the distribution of π is given by the standard exponential distribution and the distribution of c is given by the exponential distribution $f_s(c) = e^{-(\mu-c)}$, $c \leq \mu$, and $\mu > 0$.

18. Minimizing the aggregate disagreement payoff would not always be optimal if the social welfare function had an interior maximum. Although I cannot rule out this possibility theoretically I have not found an example in which this actually is the case. For all the distributions that I have considered, the social welfare function is quasi-convex so that $\underline{d}$ or $\overline{d}$ are always optimal.

19. See also Hart (1995, p.48).

20. Rajan and Zingales (1998) show that joint ownership can be optimal in a property rights model if the investments reduce the players' outside options. Halonen (1995) shows that in an infinitely repeated game with ex ante investments joint ownership can be optimal for reputational reasons. Also, de Meza and Lockwood (1998) and Chiu (1998) show that joint ownership can be optimal when ex post bargaining takes the form of a Rubinstein alternating offers game with outside options.

21. For a discussion of the empirical evidence of the property rights theory see Whinston (2000).

22. In particular, I extend Admati and Perry (1987) by allowing for non-zero inside options. They consider a Rubinstein (1982) alternating offers game in which each player, after having received an offer, can decide after how much time to make a counter-offer.

23. For more details see www.britannica.com.

6 Appendix

Proof of Proposition 1. For the purpose of this proof it is useful to introduce the following definitions: $\widehat{p}_s(c) \equiv \frac{1}{12}(3 + 9s(\cdot) - 3b(\cdot)) + \frac{2}{3}c$, $\widehat{p}_b(\pi) \equiv \frac{1}{12}(1 - 9b(\cdot) + 3s(\cdot)) + \frac{2}{3}\pi$, $x \equiv \max[\widehat{p}_s(0), p_b(0)]$, and $y \equiv \min[p_s(1), \widehat{p}_b(1)]$.

I first verify that $p_b(\pi)$ is a best response to $p_s(c)$ and then the reverse. If the seller plays $p_s(c)$, the buyer's best response must solve

$$\max_{p_b} B(p_b, \pi), \tag{6}$$

where

$$B(p_b, \pi) \equiv b(\cdot) + \left[\pi - b(\cdot) - \frac{1}{2}(p_b + E(p_s(c) \mid p_s(c) \leq p_b))\right] \text{prob}(p_s(c) \leq p_b),$$

$$\text{prob}(p_s(c) \leq p_b) = \begin{cases} 0 & \text{if } p_b < x \\ \frac{3}{2}(p_b - \widehat{p}_s(0)) & \text{if } x \leq p_b \leq p_s(1) \\ 1 & \text{if } p_b > p_s(1), \end{cases}$$

and

$$E(p_s(c) \mid p_s(c) \leq p_b) = \begin{cases} \frac{1}{p_b - \widehat{p}_s(0)}\left[\frac{1}{2}x^2 + \frac{1}{2}p_b^2 - \widehat{p}_s(0)x\right] & \text{if } x \leq p_b \leq p_s(1) \\ \frac{3}{2}\left[\frac{1}{2}x^2 + \frac{1}{2}p_s(1)^2 - \widehat{p}_s(0)x\right] & \text{if } p_b > p_s(1). \end{cases}$$

Note that it can never be optimal to set $p_b > p_s(1)$ since $B(p_b, \pi) < B(p_s(1), \pi)$ for any $p_b > p_s(1)$. Also, it can never be strictly optimal to set $p_b < x$ since $B(p_b, \pi) \leq B(x, \pi)$ for any $p_b < x$. Finally, note that $B(p_b, \pi)$ is strictly concave in p_b for any $x \leq p_b \leq p_s(1)$. The first order conditions for the buyer's maximization problem are therefore given by

$$\frac{\partial B(p_b, \pi)}{\partial p_b} \leq 0 \text{ if } p_b = x, \quad \frac{\partial B(p_b, \pi)}{\partial p_b} = 0 \text{ if } x < p_b < p_s(1), \text{ and}$$

$$\frac{\partial B(p_b, \pi)}{\partial p_b} \geq 0 \text{ if } p_b = p_s(1).$$

This implies that the buyer's best response is given by $p_b(\pi) = \widehat{p}_b(\pi)$ for any $\pi \leq \frac{1}{4}(5 + 3d(A))$ and $p_b(\pi) = p_s(1)$ for any $\pi > \frac{1}{4}(5 + 3d(A))$.

Next I verify that $p_s(c)$ is a best response to $p_b(\pi)$. If the buyer plays $p_b(\pi)$, the seller's best response must solve

$$\max_{p_s} S(p_s, c),\tag{7}$$

where

$$S(p_s, c) \equiv s(\cdot) + \left[\frac{1}{2}\left(p_s + E\left(p_b(\pi) \mid p_b(\pi) \geq p_s\right)\right) - c - s(\cdot)\right]\mathrm{prob}\left(p_b(\pi) \geq p_s\right),$$

$$\mathrm{prob}\left(p_b(\pi) \geq p_s\right) = \begin{cases} 0 & \text{if } p_s > y \\ \frac{3}{2}\left(\widehat{p}_b(1) - p_s\right) & \text{if } p_b(0) \leq p_s \leq y \\ 1 & \text{if } p_s < p_b(0), \end{cases}$$

and

$$E\left(p_b(\pi) \mid p_b(\pi) \geq p_s\right) = \begin{cases} \frac{1}{\widehat{p}_b(1) - p_s}\left[y\widehat{p}_b(1) - \frac{1}{2}y^2 - \frac{1}{2}p_s^2\right] & \text{if } p_b(0) \leq p_s \leq y \\ \frac{3}{2}\left[y\widehat{p}_b(1) - \frac{1}{2}y^2 - \frac{1}{2}p_b(0)^2\right] & \text{if } p_s < p_b(0). \end{cases}$$

Note that it can never be strictly optimal to set $p_s > y$ since $S(p_s, c) \leq S(y, c)$ for any $p_s > y$. Note also that it can never be optimal to set $p_s < p_b(0)$ since $S(p_s, c) < S(p_b(0), c)$ for any $p_s < p_b(0)$. Finally, note that $S(p_s, c)$ is strictly concave in p_s for any $p_b(0) \leq p_s \leq y$. The first order conditions for the seller's maximization problem are therefore given by

$$\frac{\partial S(p_s, \pi)}{\partial p_s} \leq 0 \text{ if } p_s = p_b(0), \quad \frac{\partial S(p_s, \pi)}{\partial p_s} = 0 \text{ if } p_b(0) < p_s < y, \text{ and}$$

$$\frac{\partial S(p_s, \pi)}{\partial p_s} \geq 0 \text{ if } p_s = y.$$

This implies that a best response is given by $p_s(c) = \widehat{p}_s(c)$ for any $c \geq -\frac{1}{4}(1 + 3d(A))$ and $p_s(c) = p_b(0)$ for any $c < -\frac{1}{4}(1 + 3d(A))$. ∎

References

[1] **Admati, A. and Perry, M.** (1987), "Strategic Delay in Bargaining," Review of Economic Studies, LIV: 345-64.

[2] **Aghion, P. and Tirole, J.** (1997), "Formal and Real Authority in Organizations," Journal of Political Economy, 105(1): 1-29.

[3] **Arrow, K.** (1975), "Vertical Integration and Communication," Bell Journal of Economics, 6(1): 173-83.

[4] **Ayres, I. and Talley, E.** (1995), "Solomonic Bargaining: Dividing a Legal Entitlement To Facilitate Coasean Trade," Yale Law Journal, 104(5), 1027-1117.

[5] **Baker, G., Gibbons, R., and Murphy, K.** (1999), "Informal Authority in Organizations," Journal of Law, Economics & Organizations, 15(1): 56-73.

[6] **Baker, G., Gibbons, R., and Murphy, K.** (2002), "Relational Contracts in Strategic Alliances," mimeo (Harvard University).

[7] **Chatterjee, K. and Samuelson, W.** (1983), "Bargaining under Incomplete Information," Operations Research, 31(5): 835-851.

[8] **Chiu, S.** (1998), "Noncooperative Bargaining, Hostages, and Optimal Asset Ownership," American Economic Review, 88(4): 882-901.

[9] **Coase, R.** (1937), "The Nature of the Firm," Economica, 4: 386-405.

[10] **Cramton, P., Gibbons, R., and Klemperer, P.** (1987), "Dissolving a Partnership Efficiently," Econometrica 55(3): 615-632.

[11] **d'Aspremont, C. and Gérard-Varet, L.** (1979), "Incentives and Incomplete Information," Journal of Public Economics, 11: 25-45.

[12] **de Meza, D. and Lockwood, B.** (1998), "Does Asset Ownership Always Motivate Managers? Outside Options and the Property Rights Theory of the Firm," Quarterly Journal of Economics, 113(2): 361-86.

[13] **Dessein, W.** (2002), "Authority and Communication in Organizations," Review of Economic Studies, 69(4), 811-838.

[14] **Grossman, S. and Hart, O.** (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," Journal of Political Economy, 94: 691-719.

[15] **Hagedorn, M.** (2003), "Contracting with Private Information," mimeo (University of Bonn).

[16] **Halonen, M.** (1995), "Reputation and Allocation of Ownership," STICERD Working Paper TE/95/289.

[17] **Hart, O.** (1995), *Firms, Contracts and Financial Structure*, Oxford University Press.

[18] **Hart, O. and Holmström, B.** (2002), "A Theory of Firm Scope," mimeo (Harvard University).

[19] **Hart, O. and Moore, J.** (1990), "Property Rights and the Nature of the Firm," Journal of Political Economy, 98: 1119-58.

[20] **Hart, O. and Moore, J.** (1998), "Cooperatives versus Outside Ownership," NBER Working Paper 6421.

[21] **Holmström, B.** (1999), "The Firm as a Subeconomy," Journal of Law, Economics, and Organization, 15(1): 74-102.

[22] **Holmström, B. and Roberts, J.** (1998), "The Boundaries of the Firm Revisited," Journal of Economic Perspectives, 12(4): 73-94.

[23] **Matouschek, N.** (2000), "A Property Rights Theory of the Firm with Private Information," in *Efficient Organisation of Economic Institutions: Firms and Contract Enforcement Agencies*, PhD Thesis (LSE).

[24] **Matouschek, N.** (2002), "Information and the Optimal Ownership Structure of Firms," CSIO Working Paper 0028.

[25] **Myerson, R.** (1991), *Game Theory*, Harvard University Press.

[26] **Myerson, R. and Satterthwaite, M.** (1983), "Efficient Mechanisms for Bilateral Trading," Journal of Economic Theory, 28: 265-281.

[27] **Rajan, R. and Zingales, L.** (1998), "Power in a Theory of the Firm," Quarterly Journal of Economics, 386-432.

[28] **Riordan, M.** (1990), "What is Vertical Integration?" in M. Aoki, B. Gustafsson, and O. Williamson (eds.), *The Firm as a Nexus of Treaties,* London: Sage, 94-111.

[29] **Rubinstein, A.** (1982), "Perfect Equilibrium in a Bargaining Model," Econometrica, 50: 97-109.

[30] **Schelling, T.** (1960), *The Strategy of Conflict*, Harvard University Press.

[31] **Whinston, M.** (2000), "On the Transaction Cost Determinants of Vertical Integration," mimeo (Northwestern University).

[32] **Williamson, O.** (1975), *Markets and Hierarchies*, The Free Press.

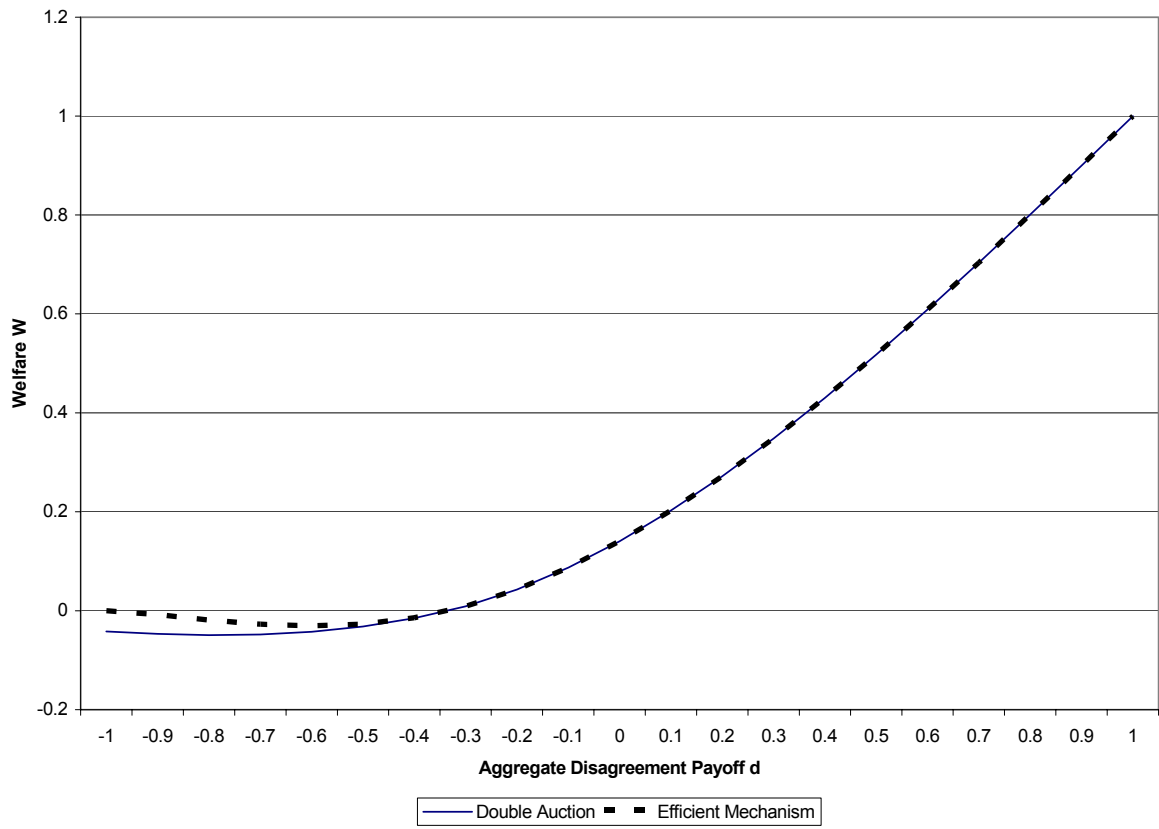[33] **Williamson, O.** (1985), *The Institutions of Capitalism*, The Free Press.

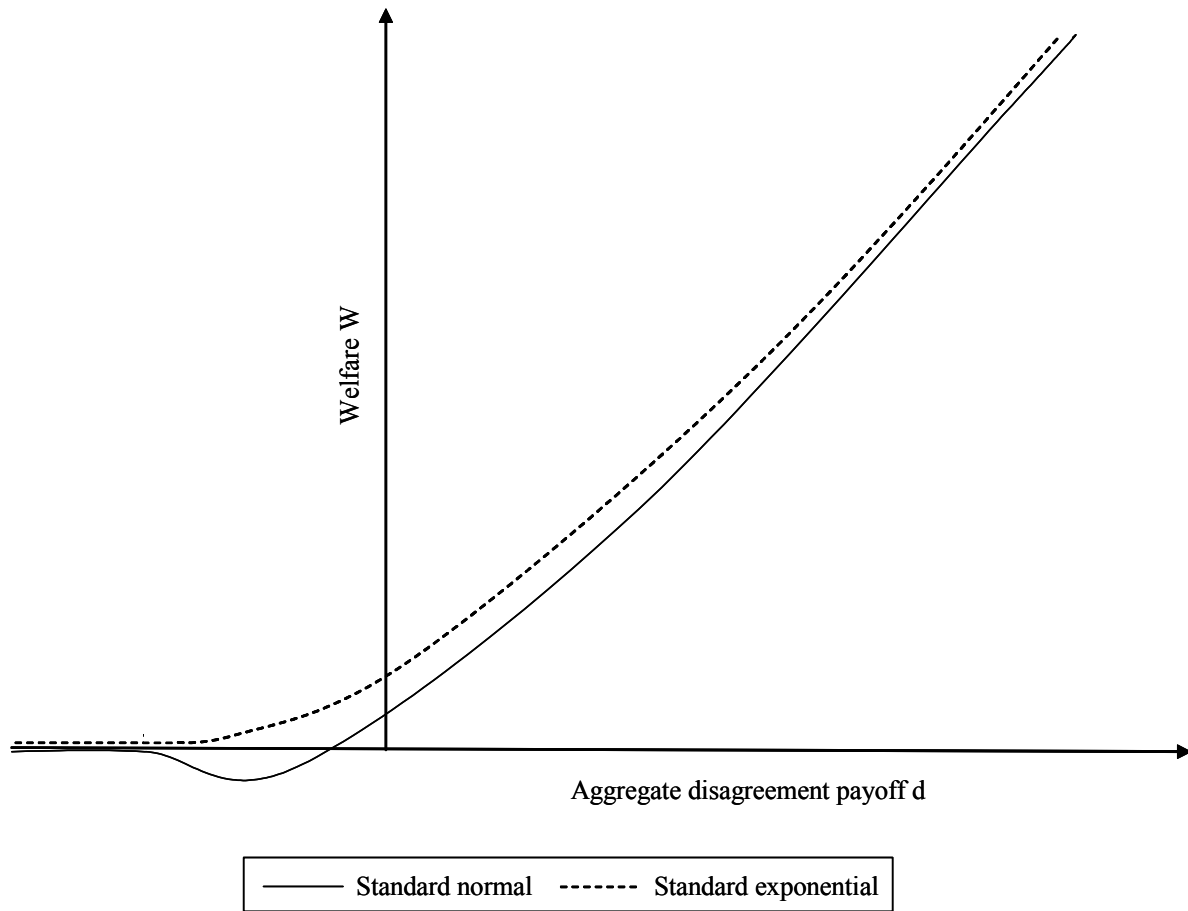Figure 1: Expected social welfare with uniform distributions

Figure 2: Sketch of expected social welfare with standard normal and standard exponential distributions